UEB TECHNICAL

Algebra and Geometry – Basic

presented by Wendy Benckhuysen, *Committee Vice-Chair* November 20, 2020 Online, Virtual Conference



NATIONAL BRAILLE ASSOCIATION

95 Allens Creek Road, Building 1, Suite 202, Rochester, NY 14618

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Introduction

The goal of this workshop is to draw attention to and clarify some sections of the Rules of UEB (including the Guidelines for Technical Material) that often come into play when transcribing algebra and geometry material.

First, we will tease out just 5 concepts, as outlined in this document's table of contents.

Next, we will study GTM Section 3, "Signs of Operation and Comparison," a new version of which was published on the <u>UEB page of ICEB's website</u> in October 2018 (ICEB is an abbreviation for International Council on English Braille).

This workshop is not intended to be a certification-style course in transcribing. It is intended to be a support to the study of the primary resources.

This workshop does not replace or supersede any BANA or ICEB publication.



Superscripts

A superscript is a letter, figure, or symbol written or printed above the line. (Oxford) Since we cannot change the physical shape or vertical position of braille, we use braille symbols to give information about print line position. Some of the braille symbols we use to do this are shown below.

Superscript (level change up)

Opening braille grouping indicator

Closing braille grouping indicator

(The dot locator for mention should precede each symbol on a Special Symbols Page; this can help a braille reader to figure out where the listed symbol falls within a braille cell.)

In other words, if we put the "Superscript (level change up)" indicator before a number, we are saying "it is a superscript" and "it is printed above the line."

I would rather win 10^{10} dollars than 10^{2} .

The numeric indicator must be repeated after the superscript indicator, because numeric mode cannot continue over a level change indicator. (RUEB §6.3.1)

 $E = mc^2$

To get the grade 1 effects where necessary, the transcriber chose to use a grade 1 passage. This expression could also be transcribed

: and "in" – grade 1 indicators

The superscript indicator has the same dot configuration as the lower wordsign and lower groupsign "in." So, what's a transcriber to do?

We must make sure to use the superscript indicator in grade 1 mode. Otherwise, it will mean "in".

The numeric indicator establishes grade 1 mode.

 $5^{x} 10^{8}$

Grade 1 mode can also be established by using grade 1 indicators, shown below.

More on choosing which way of establishing grade 1 mode will come up as we explore more examples in this workshop.



Scope of a Superscript Indicator

Like other braille indicators, the superscript indicator's effect has limits.

The amount of material on which a superscript can have an effect – its scope of influence, if you will – is defined in GTM section 7.1.

A superscript indicator can affect an item.

An item is defined, briefly, as any of the following if they come immediately after the level change indicator:

- 1. An entire number, which includes everything in the established numeric mode (e.g., interior decimal points, commas, separator spaces, and simple numeric fraction lines).
 - a. These are all examples of entire numbers: 5 42 3.14159 ½ 5,280 3.1415926535 8979323846 2643383279
- 2. An entire general fraction, enclosed in fraction indicators.
- 3. An entire radical expression, enclosed in radical indicators.
- 4. An arrow.
- 5. An arbitrary shape.
- 6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces.
- 7. Any expression enclosed in the braille grouping indicators.
- 8. If none of the foregoing apply, the item is simply the next individual symbol.
 - a. These are all examples of individual symbols: $* = \infty$

How to make your superscript last longer

We can expand the scope of influence of a superscript by using braille grouping indicators. So, when something (or a string of somethings) is "superscripted" and is not an item by any of the other definitions above, braille grouping indicators can make it (or them) into an item.

x^2

Without any braille grouping indicators, the braille says, "x superscript minus ... and then two."

x^{-2}

With braille grouping indicators, the braille says, "x superscript minus-two."

This applies to SUBSCRIPTS, TOO!

The technique for making a superscript last longer works just the same for subscripts (and also for common modifiers).

$x_2 + k$

Without any braille grouping indicators, the braille says, "x subscript two ... and then plus k."

A grade 1 symbol indicator is only needed within the first three symbols of the symbols-sequence, so the transcriber chose to use that solitary grade 1 symbol indicator early on (and not use a grade 1 word indicator for the whole symbols-sequence). (#5 in BANA's <u>Provisional Guidance for Transcribing Mathematics in UEB</u>)

*x*_{2+k}

With braille grouping indicators, the braille says, "x subscript two-plus-k."

Both the subscript indicator and the opening braille grouping indicator need to be in grade 1 mode, so the transcriber chose to use a grade 1 word indicator for the whole symbols-sequence (instead of using two grade 1 symbol indicators, one for the \vdots and another for the \vdots). (#5 in BANA's Provisional Guidance for Transcribing Mathematics in UEB)

Δ regular polygon

Please note that no contractions are used in the word "regular," because grade 1 mode was set earlier in the symbols-sequence. The closing braille grouping indicator after the word "polygon" has a grade 1 symbol indicator, because the space before the word terminated grade 1 mode [to the best of our understanding].

This could also be transcribed:

Braille grouping indicators are also used for multiple levels of superscripts, subscripts, and modifiers as well as for multi-word descriptions of a transcriber-defined shape, but those uses are outside the scope of this workshop.



Common Modifiers

Common modifiers such as the bar, arrow, dot, tilde, hat or arc are treated separately from superscripts and subscripts. (They're so common/frequent that they have special dedicated symbols all their own.) These common modifiers are covered in Section 12 of the GTM.

We are going to focus in this workshop only on common modifiers that are *above* the item they modify. There is a whole set of common modifiers that go under the item they modify, but they are outside the scope of this basic workshop.

Bar over previo	us item	•											
Simple right-po over previou	-												
Dot over previo	ous item												
Tilde over previ	ous item												
Hat over previo	us item												
Arc over previo	us item												
Opening braille indicator	grouping	•											
Closing braille g indicator	grouping	•											
	(On a Special	Symbols Page, 1	the dot locato	or for mention	should precede each	ı symbol.)							
$\overline{\mathrm{B}}$	\vec{G}	<u>.</u>	ĩ	\hat{y}	Â								
bar	arrow	dot	tilde	hat	arc								

Section 12 of the Guidelines for Technical Material gives a full list of the braille symbols for common modifiers, including those that are *below* the item they modify.

Scope of a Common Modifier

The effect of a common modifier applies only to the **item** that immediately precedes it. The definition of an item to which a common modifier applies is the same as the definition of an **item** that determines the <u>scope of a</u> <u>superscript</u>. How convenient!

If the common modifier applies to something more or less than the item that precedes it, then the scope of the common modifier is changed by using braille grouping indicators.

This can be particularly pertinent in geometry for things like:

 \overrightarrow{EF} arrow over EF

The braille grouping indicators make the simple right-pointing arrow apply to the E and F together instead of only to the F.

BC "BC bar" or bar over BC

The braille grouping indicators make the bar apply to the B and C together instead of only to the C.

A grade 1 word indicator precedes the symbols-sequence is in the symbols sequence is in the symbols sequence

BA arc BA

The braille grouping indicators make the arc apply to both the B and A instead of just the A.

A grade 1 word indicator precedes the symbols-sequence is is in the second sequence is is in the second sequence is in the



1. $\dot{3}$ 1.3 with a dot over the 3

The braille grouping indicators make the dot apply to just the 3 instead of to the entire number 1.3.

No grade 1 indicators are used, because the numeric indicator at the beginning of the symbols-sequence sets grade 1 mode for the whole thing.

A numeric indicator is used before the 3, because the opening braille grouping indicator terminates the effect of the numeric indicator before the 1.

"Bars and dots etc." and grade 1 indicators

As we have seen in the examples above, grade 1 indicators are often necessary when braille grouping indicators are involved, because braille grouping indicators correspond to the contractions for "gh" and "ar."

In addition, most of the common modifiers have a grade 2 meaning, so grade 1 mode must be established so they are not misread.

Line Notation

There is one notation that is common in geometry that UEB does not classify with the "common modifiers." That is the "bidirectional arrow over," which can denote a line.

Ă

The last four braille cells above make up the common horizontal bidirectional arrow.

The two braille cells before the common horizontal bidirectional arrow is the symbol for "expression directly above."

No braille grouping indicators are necessary, because the single letter A is an item, and that is what the modifier affects.

A grade 1 word indicator precedes the symbols-sequence is in the symbols because otherwise it would mean "A with "ouwro" over it."

ĂŻ

The braille grouping indicators make the bidirectional arrow apply to both the A and Z instead of just the Z.



Shape Symbols and the Shape Terminator

Now that we have touched on line notation, common modifiers, and superscripts, let's talk about shapes.

Geometry material uses a *lot* of shapes. Boy-oh-boy, can we ever *not* draw little shapes within the braille line. So, we use braille symbols to give information about the little shape that appears in print.

Initial Shape Indicators

The kind of information we give about a shape is basically "what shape is it?" and "what do its insides look like?" The second question (what do its insides look like?) is answered by the initial shape indicator used.

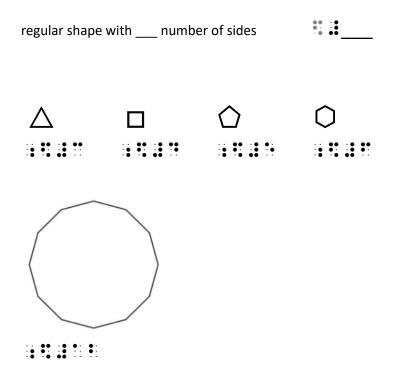
Shape indicator	••
Filled (solid) shape indicator	
Shaded shape indicator	

Specific Shape Symbols

The first question (what shape is it?) is answered by what comes after the initial shape indicator. UEB gives us specific symbols for a parallelogram, a circle, and any shape with sides all the same length.

parallelogra	am			••		
circle						What is a parallelogram? a flat shape with opposite sides parallel and equal in length -mathisfun.com
		7		7	L	
The first ite	m above is a	solid (filled)	shape		that is a parallelogram	
The second	item above	is a shaded s	hape		that is a parallelogram	
The third it	em above is	a [hollow] sh	аре		that is a parallelogram	
lacksquare	۲	0				

For any **shape with sides all the same length** (also called "regular" shapes), we answer the question "what shape is it?" with a number that corresponds to the number of sides that make up the shape. This includes squares \Box , equilateral triangles \triangle , hexagons \bigcirc , etc.



All sides of a \blacksquare are the same length.																											
• •	•••	۰ ě	•		•				•			- · •	÷.	- · ě				•	•	•••		÷.		••	÷÷.	÷.	••



H and "ed" – grade 1 indicators

The initial shape indicator for a shape that is hollow has the same dot configuration as the strong groupsign "ed." So, what's a transcriber to do?

We must make sure to use the initial shape indicator for a shape that is hollow in **grade 1 mode**. Otherwise, it will mean "ed."

The numeric indicator establishes grade 1 mode. The mode can also be established by grade 1 indicators (i.e., the grade 1 symbol, word, and passage indicators).

Label the \triangle as **equilateral**, **right**, **scalene**, or **isosceles**.

The filled shape indicator and the shaded shape indicator cannot be anything other than initial shape indicators, regardless of whether they are in grade 1 or grade 2 mode. So, grade 1 mode does not need to be forced for either is or is is.

Place a • at coordinates (3, 4).

Shape Terminator

We use one of the initial shape indicators to establish that we are giving information about a shape. But how is a reader to know when we have stopped giving information about that shape and moved on to other things?

Shape terminator

The shape terminator tells them when it is over.

•

USE it when ... another symbol follows immediately after

If the shape symbol is followed by punctuation or is unspaced from a following symbol, then the shape terminator must be used. (GTM 14.1.2)

```
 + x = 52
```

Is it a \triangle , \square , or \bigcirc ?



Leave it OFF when ... a blank cell defines the scope of the shape indicator

If a shape is followed by a space, then no termination symbol is needed. (GTM part 14.1.1)

 $x + \square = 52$

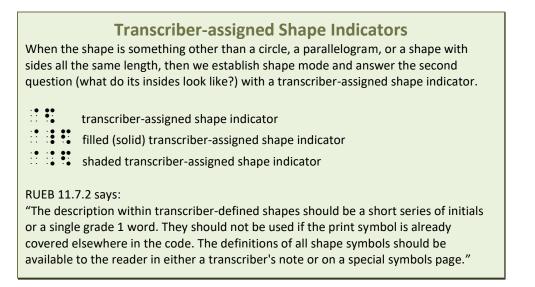
△ ABC

A \triangle has three sides.

H and *Shape Mode* – no grade 1 indicators

All the initial shape indicators initiate shape mode so no further grade 1 indicators will be needed. (note to GTM 14.1.2)

In other words, once shape mode is established, we don't have to worry about setting grade 1 mode, because it is already established.



Shapes for Measurement or Identification

Rules of UEB (RUEB) §16.1.3 says, "Do not use line mode **when the attributes of the lines or their relationship with one another is important**, such as in the study of geometry or the measurement of angles." (emphasis added)

Similarly, when the measurable attributes of a shape and/or its relationship to others is the reason the shape is included in the text, *then the shape indicators just discussed are not appropriate*.

For example, if the student is expected to

- count the sides of a shape in order to identify it
- identify a triangle as equilateral, isosceles, or scalene
- study the measurement of an angle
- find the coordinates of a shape

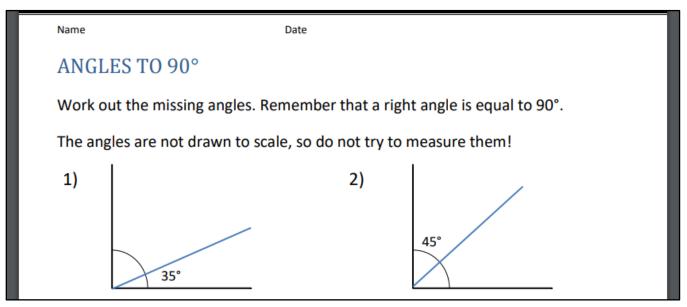
then a tactile graphic is a better option than either shape indicators or line mode.

Again, when the measurable attributes of a shape and/or its relationship to others is the reason the shape is included in the text, the student or reader needs a detailed, measurable representation of the shape in order to study its attributes.

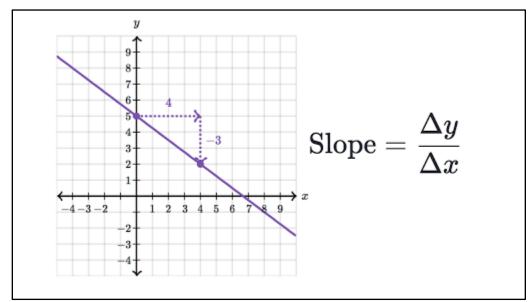


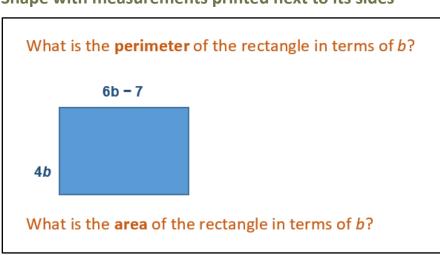
Some Examples for Discussion

Angles with measurements



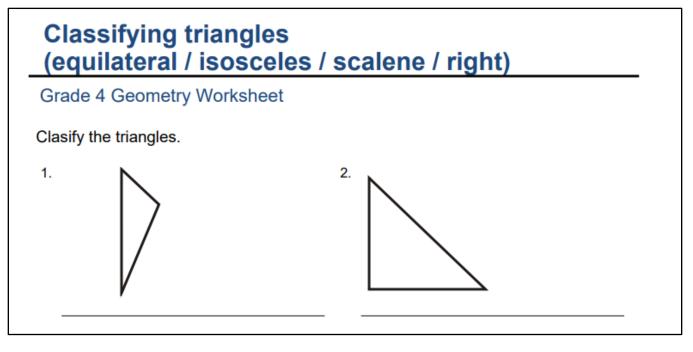
Plotted line





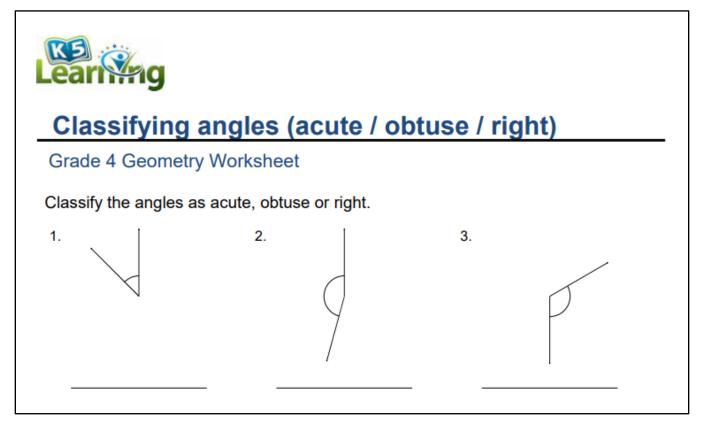
Shape with measurements printed next to its sides

Classifying triangles





Classifying angles



"Where the technology is available, it is often better to represent lines in diagrams with tactile graphics." –RUEB §16.1.5

The Beauty of Print

Formatting is not what this workshop is about, so we will not go into great depth about formatting. Nonetheless, we acknowledge that one of the first steps to a successful transcription is recognizing what is nothing more than embellishment and choosing to ignore it. Let's touch on a few guiding principles when it comes to the "siren song" of beautiful print.



1 painting "Ulysses and the Sirens" 1891, John William Waterhouse

Ignore Typeform for Variables

We **only** retain typeform (italics, bold, etc.) for a variable if it distinguishes the variable from another variable. For practical purposes, italicization is never retained for variables.

In the equation y = mx + b, what is the meaning of the y?

A vector is often written in **bold**, like **a** or **b**.

Typeforms are also discussed in the 15-minute webinar "Typeforms" in the series of webinars on UEB Technical Material, which is part of a <u>webinar archive</u> that is free to NBA members.



Follow BANA Formatting Guidelines

The formatting guidelines for a UEB Math/Science transcription in the U.S. and Canada come from Braille Formats 2016 and BANA's "Provisional Guidance for Transcribing Mathematics in UEB."

Below is a brief summary of the formatting guidance given in that BANA document.

- 3-1 (aka indented) paragraphs only
- Displayed literary text is formatted according to BF2016, except for paragraph format
- Mathematical expressions are displayed according to BF2016, using the formula:
 - blank line
 - add two to preceding runover position
 - add two more for displayed's runover position
 - blank line
- 1-3 can be used for a big expression in higher math
- Directions use margins 5-5, 7-5
- Exercises are formatted according to BF2016 (nested list and format determined for full set)
- Abbreviations stay on a braille line with their related number

BANA's "Provisional Guidance for Transcribing Mathematics in UEB" is completely discussed in the NBA webinar <u>UEB Provisional Guidance Walkthrough</u> that is for sale on NBA's website.

Other Considerations

Spacing

Generally, space signs of comparison and unspace signs of operation. If it helps, we can think of this as opening up some distance (in order to have "perspective") when things are compared and closing the space to get proximity for performing an operation (like a surgical operation).

x + y = z $90^{\circ} - 60^{\circ} = 30^{\circ}$ $\overline{AD} + \overline{BC} = \overline{AC}$ $\overline{AD} + \overline{BC} = \overline{AC}$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\overline{AD} = \overline{AC}$

The "why" of this spacing directive is touched on in the *Note* at the beginning of the latest section 3.1 in GTM. It says, "Note: The presence or absence of spaces in braille is an important aid to parsing mathematical expressions and equations. Print spacing is often simply a matter of printing style."

Bluntly, spacing is more important in braille than it is in print. So, stay consistent in braille, regardless of print's stylistic spacing choices.



Blanks

Follow print, using the least complex equivalent you can get away with. For example, use one underscore instead of a certain length of underscoring (assuming print is not using a specific number of separate underscores). Similarly, instead of transcribing a "blue rectangle," use a rectangle (assuming there are not multiple colors of rectangle).

Fill in the blank: y = mx + ?Fill in the blank: y = mx +_____

Insert the three missing numbers: 3.14___

A quote from the recent ICEB General Assembly, "Yes, boxes are clunky." ... And, *if they are necessary*, we use them, regardless of clunkiness.

Formatting is also discussed in the 15-minute webinar "UEB Technical - Format" in the series of webinars on UEB Technical Material, which is part of a <u>webinar archive</u> that is free to NBA members.

GTM Section 3: Signs of Operation and Comparison

The most current version of Section 3 in the Guidelines for Technical Material (GTM) was approved in October 2018. It is the first completed section of a revised GTM. A full list of updates to UEB rules can always be found at http://iceb.org/ueb.html.

Let's review the highlights of GTM Section 3, piece by piece.

Operation Signs & Comparison Signs

Section 3 provides two informative tables at its very beginning. These tables not only list signs of operation & comparison and their braille symbols; they also give:

- 1) a copy/pastable character for each sign (not just a picture of the print symbol), so you could paste it right into the print input area of a braille transcription program and get a useful braille transcription
- 2) the numeric Unicode value for each sign, and
- 3) a variety of names for each sign (so if you search the file you are more likely to find what you are looking for on the first try and so you could use a search engine to find more information about a sign).

What is a Unicode value, and how can I use it?

Unicode is an international encoding standard for use with different languages and scripts, in which each letter, digit, or symbol is assigned a unique numeric value that applies across different platforms and programs.

Bluntly, Unicode is a system in which a specific number corresponds to a very specific symbol.

A person can use a Unicode number to enter a symbol in a variety of programs.



Spacing

In this workshop, we have already touched on the concept of spacing. The GTM (Guidelines for Technical Material) basically says, in the absence of other instruction, unspace symbols of operation, and space symbols of comparison.

When in doubt about whether a sign is a symbol of operation or comparison, look to the tables at the beginning of GTM Section 3 – one is full of signs of operation and the other is full of signs of comparison!

12.5y = 7 - x	$63 = x \times 9$								
$y \neq 0$	101 > <i>z</i>								
45° + ? = 90°	AB WX								

Do what is best for the student

The Guidelines say, "Signs of operation may be spaced when they are first taught, before transitioning to normal spacing practice." Signs of operation MAY be spaced when a student is first learning them. Again and as always, do what is best for the student if you can find that out.

A spaced sign of operation would likely make more numeric and grade 1 indicators necessary.

It is unlikely that a student studying algebra or geometry is first learning signs of operation.

Spacing is also discussed in the 15-minute webinar "UEB Technical - Spacing" in the series of webinars on UEB Technical Material, which is part of a <u>webinar archive</u> that is free to NBA members.

That webinar includes discussion of exceptions for spacing signs of comparison when 1) they appear in an expression that is not on the base line and 2) to avoid dividing an expression between braille lines.

3.1.5

"Signs of operation may be spaced to enhance the parsing of an expression which includes spaces between quantities and their units."

$\ell \operatorname{ft} \times w \operatorname{ft} \times h \operatorname{ft} = A \operatorname{ft}^{3}$

The abbreviated units of measurement (ft) are spaced away from both the variable to which they apply and also from the sign of operation that follows; this might make the equation easier to parse (or analyze).

3.1.6

"Follow print spacing for signs of operation and comparison when the adjacent text is not a wholly mathematical expression."

In other words, the directive to space signs of comparison and unspace signs of operation is meant to apply within mathematical expressions, not within narrative sentences.

Joseph has ± 5 acres for sale.

It would not be mathematically accurate, nor would it be helpful, to have the ± unspaced from the word "has" in this narrative sentence.

Minus

3.2's Note

A minus in print looks a lot like a hyphen.

fifty-five – twenty-two = thirty-three

In *excellently* formatted print, an actual minus sign may have a different Unicode value (usually 2212) than a hyphen (usually 2010 or 002D [technically a "hyphen-minus"]).



3.2.1

"It is permissible to use a braille hyphen for a minus which is indistinguishable from a hyphen in print."

In other words, if you cannot tell the difference from the print, then it is better to make no distinction between the hyphens and minuses than to interpret the material and impose your own distinction for the braille user.

one-time-opportunity - fear = success is as simple as 2+2 = 4.

Positive and Negative Numbers

3.3.1

"Use a superscript indicator when a plus or minus, indicating a positive or negative number, is in the superscript position."

4 - -2 = 6

The minus sign that is superscripted in print is also in the superscript position in braille.

This expression does not require any grade 1 indicators, because numeric mode sets grade 1 mode.

$0 - 32 = -32^{\circ} F$

The minus sign in the superscript position before the second 32 is brailled in the superscript position.

The superscript indicator after a space is not in numeric mode; it needs to have grade 1 mode applied to it. So, the transcriber chose to use a grade 1 symbol indicator before the superscript indicator that follows a space.

3.3.1's Note

"It is permissible to describe the superscript position in a transcriber's note instead of using a superscript indicator."

If the superscript position does not add any information about the sign of operation, then it **may** be ignored in braille.

Jane said, "I cannot abide water above +16°C."

"That's nothing. I only take my water supercooled to -48.3°C," replied John.

A transcriber's note before the text says, "In the following paragraphs, print uses a superscript plus for a positive number and a superscript minus for a negative number. The superscript position is not shown in braille."



The Hollow Dot

3.4.1

"Use the hollow dot to represent the mathematical sign of operation."

The hollow dot is not an unshaded, unfilled circle, like the one used as a bullet. Nor is the hollow dot the degree sign.

Consider the following list:

- 27° C is darned hot.
- $(f \circ g)(x)$ can be said as "f-compose-g of x."

					• •	
				••	•	
• • • •	• • • •		 	 		· •

At the beginning of each listed item, print is followed for the bullets, which are hollow circles.

After the 27, the degree symbol is used for the printed degree sign.

In the math expression, the ring operator (aka hollow dot) is used for the hollow dot in the function.

The Asterisk

3.5.1

"Use the braille asterisk to represent the print midline asterisk used as a sign of operation in mathematics. It is generally brailled unspaced."

The midline asterisk sign of operation is not the same sign as the asterisk used to mark notes, indicate emphasis, etc. ... though both symbols do have the same braille symbols.

"I am *not* ready yet. I need 3*3 (or 9) hours to finish getting ready!" said Carla*

*Carla is not a patient person.



Concepts for Algebra and Geometry

- 1. Superscripts and similar indicators (subscripts, modifiers directly over/under, and common modifiers) can have the scope of their influence adjusted with braille grouping indicators.
- 2. Common modifiers have their own specific symbols and are covered in section 12 of the GTM.
- 3. Shape symbols work by answering two questions: "what shape is it?" and "what do its insides look like?"
- 4. Shapes for measurement or identification should not be transcribed using shape symbols.
- 5. We must not fall prey to the siren song of the beauty of print. We ignore typeform for practically all variables, and we are consistent with our spacing even when print is not.
- 6. Grade 1 must be used to keep symbols from being misread. How that mode is applied varies based on the expression.

Closing Words

Thank you for spending some of your very valuable time and attention on this workshop.

End of the material for Algebra and Geometry – Basic (UEB Technical)