Mathematics and Science Braille
Nemeth in UEB

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Introduction

We are going to explore sections of the guidance in the order laid out in the document posted on the website of Braille Authority of North America (BANA).

The first and overriding principle is that, within a UEB document, mathematical and technical expressions will be transcribed in Nemeth Code. This is accomplished by setting apart the technical material with the use of the Opening Nemeth Indicator and the Nemeth Terminator. **No contractions are permitted within these switches.** Text outside of the switches is transcribed in UEB.

The single-word switch indicator was created to allow contractions in a single word between math expressions and to avoid having to switch out of Nemeth code for one word. The switch is required on a single word even if the word contains no contractions. The effect of the single-word switch is terminated by a space. (The single-word switch indicator is a Nemeth symbol, not a UEB symbol.)

Opening Nemeth Code indicator :•

Nemeth Code Terminator ：

Single-word switch indicator ：

(The dot locator for mention : :: precedes each indicator in the list on the Special Symbols page.)
Basic Format

The switch indicators signal that the symbols used are Nemeth Code symbols, but the switches do not govern formatting. The entire document is formatted according to the established mix of Nemeth Code and Braille Formats provisions as posted on the website of Braille Authority of North America and available there for downloading. When the technical material is transcribed in Nemeth Code, the whole document is formatted according to the Nemeth Code—even the parts of the text that are not in Nemeth. We will discuss format later in the session.
Special Symbols

When material is transcribed using the Nemeth Code, and this is stated on the Transcriber's Notes page, it is not necessary to list on the Special Symbols page any symbol of the Nemeth Code. The TN page should have a note stating: "Mathematical content is transcribed according to the Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision, 2007-2015 Updates, including the guidelines for using Nemeth in a UEB context."

However, the single-word switch indicator is a new symbol and may not be recognized by the reader. It should be listed on the Special Symbols page.
Use of Opening and Closing Nemeth Indicators

Place the opening Nemeth Code indicator followed by a space before the sequence to which it applies. Its effect is terminated by the Nemeth Code terminator preceded by a space. The spaces required with the indicator and terminator do not represent spaces in print.

Example 1

Equations that represent functions are often written in a special way. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$.

Line 1: All paragraphs in a document governed by Nemeth formatting begin in cell 3 with runovers in cell 1. There are no blocked paragraphs in Nemeth.

Line 1: UEB boldface word indicator is used on a bold word that is not within the Nemeth Code indicators.

Lines 1 and 2: Note the contraction for "tion" in the word equation

Line 3, 4: Nemeth Code is initiated by the use of the Opening Nemeth indicator and ends with the Nemeth Terminator. Math expressions are transcribed according to the rules of The Nemeth Code, within the Nemeth indicators. If possible, the indicators are on the same line as the expression.

Line 4: The period applies to the structure of the sentence and not to the math expression. It follows the termination indicator.
When the Nemeth Code text is displayed on one or more lines separate from the UEB text, the Opening Nemeth indicator may be placed on a line by itself or at the end of the previous line of text; the terminator may be placed at the end of the last line of the math material or on a line by itself. The opening indicator is placed at the end of the text line preceding the math expression in preference to being on a line by itself.

**Example 2 (displayed)**

The factor by which the sound intensity must be increased to double the loudness can be determined by the method used in Example 9:

\[ \beta_2 - \beta_1 = 10.0 \text{ dB} = (10 \text{ dB}) \left[ \log \left( \frac{I_2}{I_0} \right) - \log \left( \frac{I_1}{I_0} \right) \right] \]

Solving this equation reveals that \( I_2/I_1 = 10.0 \).
Line 1: Indented paragraph according to Nemeth format.

Line 3: Single number is transcribed in UEB.

Line 3: Opening Nemeth Code Indicator is placed at the end of the text line preceding the displayed material.

Line 4-7: Displayed expression. A division must be made at every comparison sign because a division is necessary within a link.

Line 8: The Nemeth Code Terminator will not fit at the end of the last line of the displayed material. It is placed on the following line in the runover position.

Line 10: The entire technical expression and the code indicators will fit on one line. The period is related to the sentence structure, not to the math expression.
Basic Guidance on When to Switch

Those are the basics. But to apply the system to an actual text or worksheet, we need to go into a little more detail.

Any mathematical or technical notation, including fragmentary expressions, such as parts of formulas, incomplete equations, etc., should be presented in Nemeth Code, while the surrounding text is transcribed in UEB.

Example 3

At time \( t = 0 \) s automobile A is traveling at a speed of 18 m/s along a straight road and is picking up speed with an acceleration that has a magnitude of 3.5 m/s\(^2\). At time \( t = 0 \) s automobile B is traveling at a speed of 18 m/s in uniform circular motion as it negotiates a turn. It has a centripetal acceleration whose magnitude is also 3.5 m/s\(^2\). Determine the speed of each automobile when \( t = 2.0 \) s.

Line 1: The math expression is transcribed in Nemeth code within the opening Nemeth code indicator and terminator. An abbreviation must be spaced away from its related number.

Line 1, 6: Automobile identifiers A and B are transcribed in UEB.

Line 5, 6: Nemeth rules are followed regarding the division of math expressions between braille lines. The entire expression including open and close indicators is kept on one line when possible.

Line 5, 10, 12: The period relates to the sentence structure.

Line 6: "s" is an abbreviation spaced away from its related number (0)
Exceptions

Freestanding, unmodified numbers and/or letters can be transcribed in UEB. If a number or letter is combined with anything other than an ordinal ending or internal comma (e.g., a minus sign, a decimal point, etc.), it is transcribed within the switches and according to Nemeth Code.

Example 4 (numbers)

100. (a) Calculate the BACs in Exercises 98 and 99 if each person weighs about 25 lb more and the rest of the variables stay the same. How does increased weight affect a person's BAC?

(b) Predict how decreased weight would affect the BAC of each person in Exercises 98 and 99. Calculate the BACs if each person weighs about 25 lb less and the rest of the variables stay the same.

Line 1: Main item identifier is in UEB. Following Nemeth format, the main item starts in cell 1 and would run over in cell 5.
Line 2, 7: Subdivision identifiers. Each subdivision starts on a new braille line. Margins are 3-5.
Line 2: UEB parentheses; letter a does not need a grade one indicator.
Line 6: An abbreviation by itself is not mathematical and doesn't require a switch to Nemeth.
Line 7: UEB parentheses; letter b needs a grade one indicator.
Line 11: Even though "25" would fit on the line above, according to Nemeth code an abbreviation may not be separated from its applicable number. Nemeth formatting is followed in the document even when notation is not in Nemeth. (Guidance)
Example 5 (letters and numbers)

When using your graphing calculator to approximate real zeros, it is helpful to know that a function with degree \( n \) has \( n \) zeros. This is a corollary to the Fundamental Theorem of Algebra, which you will study in Lesson 10-4. Since an \( n \)th degree function has \( n \) zeros, a function with degree 5 has 5 zeros.

1. Use your graphing calculator to approximate real zeros; it is helpful to know that a function with degree \( n \) has \( n \) zeros. This is a corollary to the Fundamental Theorem of Algebra, which you will study in Lesson 10-4. Since an \( n \)th degree function has \( n \) zeros, a function with degree 5 has 5 zeros.

Line 3: A free standing variable is transcribed in UEB using the grade one indicator. Italics is disregarded when all variables in the document are printed in italics.

Line 5: Titles for Figures, Lessons, Chapters, etc. are transcribed in UEB.

Line 6: "nth degree" is transcribed in UEB.
Example 6 (exception to the exception)

31. A farmer has 20 days in which to plant corn and soybeans. The corn can be planted at a rate of 10 acres per day and the soybeans at a rate of 15 acres per day. The farm has 250 acres of land available. If the profit on corn is $30 per acre, and the profit on soybeans is $25 per acre, how much of each should the farmer plant for maximum profit?

The numbers that include dollar signs must be transcribed in Nemeth Code. They are not unmodified. The other (freestanding, unmodified) numbers are transcribed in UEB.
Example 7 Mathematical Expressions and Free Standing Numbers

73. a. Repeat the following procedure with at least five people. Write a conjecture that relates the result of the procedure to each person's birthday.

Take the number of the month of your birthday (January = 1, February = 2, … December = 12), multiply by 5, add 6, multiply this sum by 4, add 9, multiply this new sum by 5, and add the number of the day on which you were born. Finally, subtract 165.

Line 1-2: Main item 1-5, sub-item 3-5.

Line 6-14: Displayed literary material, margins 9-7

Line 7-9: The presence of a comparison sign indicates technical/mathematical material. Parentheses that enclose only material that is to be transcribed in Nemeth code may be transcribed as part of that material even if they could logically be considered as belonging to the larger sentence structure.
When punctuation that logically applies to the sentence structure appears between technical expressions, the punctuation should be transcribed in Nemeth code to avoid switching to UEB for such punctuation.

Example 8 (punctuation/single-word switch)

**EXAMPLE 3**

a. \( x + 2 = 5 \) and \( x = 3 \) are equivalent equations because both have solution set \( \{3\} \).

b. \( a - 4 = 3, \ a - 2 = 5, \) and \( a = 7 \) are equivalent equations because they all have solution set \( \{7\} \).

c. \( y + 3 = 4, \ y - 8 = 7, \) and \( y = 1 \) are equivalent equations because they all have solution set \( \{1\} \).

---

Line 1: The Opening Nemeth indicator is placed at the end of text on line 1 to allow the itemized material to all start in the same cell.

Line 4-5, 7-9: Nemeth mode is not terminated. The next item begins with technical notation. The commas and periods that appear between consecutive math expressions are transcribed in Nemeth code even though they apply to the structure of the sentence.

Line 2, 5, 8: The single-word switch indicator allows the use of the contraction in "and". "And" is not being used mathematically; it is part of the sentence structure.

Line 10: Nemeth code is terminated before the period, assuming no technical notation follows this period.
Example 9 (punctuation/single-word switch)

In Chapter 5 we discussed polynomials. Examples of polynomials include

$$3, \quad 2x, \quad x^2 + 4, \quad \text{and} \quad x^3 - 1.$$  

In this chapter we discuss rational expressions. Rational expressions can be written as quotients (fractions) of two polynomials. Examples of rational expressions include

$$\frac{3}{2x}, \quad \frac{2x}{x^2 + 4}, \quad \frac{x^2 + 4}{3}, \quad \text{and} \quad \frac{x^3 - 1}{x^2 + 4}.$$
Line 1: Numbers such as Figure numbers or Chapter numbers are done in UEB.

Line 3: The Opening Nemeth indicator is placed at the beginning of the displayed material because the math expression and the indicators will all fit on one line.

Line 3: Even though the commas between expressions pertain to the sentence structure, they are transcribed as Nemeth commas to avoid switching just for the punctuation.

Line 3: Material that is displayed to narrative text begins in cell 3 and runs over if needed in cell 5.

Line 3, 9: The single-word switch indicator allows us to use the contraction for "and".

Line 3, 10: The period at the end of the displayed material applies to the structure of the sentence.

Line 4-5: UEB bold word indicators are used on the bold words in the sentence.

Line 8: The Opening Nemeth indicator is placed at the end of the text preceding the displayed material because the indicators will not both fit on the same line as the expression.

Line 9-10: A mathematical expression that is displayed to narrative text begins in cell 3 and runs over in cell 5.

Line 9, 10: Commas are transcribed as Nemeth commas.
Basic Guidance (continued)

Sometimes it’s not just numbers or variables that require a code switch. In the following example, the symbols have technical meaning. UEB symbols should not be used. These grouping symbols are transcribed as the reader would see them when solving problems.

Example 10 (math symbols)

If the expression contains grouping symbols, such as parentheses ( ), brackets [ ], or a fraction bar, then we perform the operations inside the grouping symbols, or above and below the fraction bar, first.

1. \[ \text{If an expression has grouping symbols, such as parentheses ( ), brackets [ ], or a fraction bar, then we perform the operations inside the grouping symbols, or above and below the fraction bar, first.} \]

Line 2: The single-word switch indicator precedes the word "brackets". The technical material will all fit on one line with the open and terminate Nemeth indicators.
Additional Guidelines

Consistency should be maintained throughout a book. For symbols such as the percent sign, degree mark, and mentions of Greek letters, the transcriber should switch to Nemeth Code for such symbols (along with any related numbers) when the same symbol occurs in math notation elsewhere within the transcription, even though the local context does not constitute math notation. Symbols such as percent or degrees should be transcribed the same way throughout a document, even though it may mean many switch indicators.
Example 11 (percent)

In 2000, the number of drive-in movie screens in the United States was about 78% of the number in 1990. About how many drive-in screens were there in 1990?

\[
\frac{78}{100} = \frac{717}{n}
\]

\[
78n = 100(717)
\]

\[
\frac{78n}{78} = \frac{100(717)}{78}
\]

\[
n \approx 919
\]

There were about 919 drive-in screens in 1990. Check by estimating.

78% of 919 \( \approx \) 0.8 \( \times \) 900 = 720, which is close to 717, the number for 2000.
Line 1, 3, 12: The date (year) is not mathematical. The numbers are transcribed in UEB.

Line 3: The percent sign is seen again in the technical part of the paragraph; it is transcribed here in Nemeth Code. It will be transcribed the same whenever it appears in the document.

Line 4: The Opening Nemeth indicator is placed at the end of the line preceding the technical notation.

Line 5-8: A series of mathematical expressions is transcribed in Nemeth. The terminator is placed at the end of the last line of technical notation.

Line 9, 13: The numbers 717 and 919 are transcribed in UEB.

Line 11: The word "of" is part of the math expression. It is transcribed uncontracted within the Nemeth switches.
Example 12 (Greek letters, degrees)

Begin by drawing the angle $\theta = \pi/3$ in standard position. Because $\pi/3$ radians is $60^\circ$, you can imagine an equilateral triangle with sides of length $l$ and with $\theta$ as one of its angles.

1. Draw $\angle \theta$.
2. $\theta$ in standard position.
3. $\pi/3$ radians is $60^\circ$.
4. Imagine an equilateral triangle with sides of length $l$.
5. $\theta$ is one of its angles.

Line 2: Math notation. The entire expression including the Nemeth switch indicators will fit on one line. $\pi/3$ is a fraction. Fractions must be transcribed in Nemeth code.

Line 3: The single-word switch indicator is used for the word "is" which occurs between two technical expressions. The comma is related to the sentence structure and follows the Nemeth terminator.

Line 3: radians is the unit of measure related to $\pi/3$. Since it is part of the technical expression, no single-word switch indicator is required.

Line 3: The degree symbol is transcribed in Nemeth Code throughout the book.

Line 5: The Greek letter $\theta$ is transcribed in Nemeth Code.

Titles for figures, tables, sections, etc. are transcribed in UEB. This applies to numbers related to the title.

Section 7-3

Figure 4.4 shows a useful application of inertia.
When words are part of an equation or math expression, they are as much a part of the technical notation as are the letter variables, numbers, signs of operation, etc. The whole expression is placed inside the Nemeth switches with no contractions and is spaced as defined in the Nemeth Code.

**Example 13 (words in equations)**

A convenient way to compare densities is to use the concept of *specific gravity*. The specific gravity of a substance is its density divided by the density of a standard reference material, usually chosen to be water at 4 °C.

\[
\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at } 4\, ^\circ\text{C}} = \frac{\text{Density of substance}}{1.000 \times 10^3\, \text{kg/m}^3}
\]
Measurement units (e.g., feet, ft., min) adjacent to related numbers which are transcribed in Nemeth Code are part of the technical expression and are transcribed within the Nemeth switch indicators.

Example 14 (measurement units)

94. **Volume of a Cube**  A bottle of perfume is packaged in a box that is in the shape of a cube. Find the volume of the box if each side is 2.5 inches long.

Line 1: UEB boldface for the paragraph heading.

Line 4: Nemeth switches are used for the number containing a decimal point. Inches is a related unit enclosed within the switches.
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In a numbered or lettered series of math problems that are in Nemeth Code, leave Nemeth mode in effect for the identifiers to avoid excessive switching, even though these identifiers are not technically part of the math.

**Example 15 (identifiers)**

11. **Physics** The speed of sound through steel is 5,200 meters per second (m/s).
   This is 2,520 m/s faster than the speed of sound through silver. Write and solve an equation to find the speed of sound through silver.

Solve each equation.

12. \( d - 4 = -7 \)  
13. \( c - 34 = 20 \)  
14. \( a - 4 = -18 \)

15. **Astronomy** Earth has an average distance of \( 1.496 \times 10^8 \) kilometers from the Sun. If light travels \( 3.00 \times 10^5 \) kilometers per second, how long does it take to reach Earth?
Line 1: The identifier is done in UEB. It is followed by text. UEB bold word indicator is used on Physics.

Line 2: The math expression extends through the "fraction" at the end of the sentence. Nemeth is terminated at the beginning of the next line. A switch indicator may stand by itself.

Line 3: Math notation is used for the expression including its related abbreviation fraction.

Line 8: The begin Nemeth indicator is placed on the line preceding the math expressions. This allows all the listed exercises to begin in the same cell.

Line 9-11: The identifiers are transcribed in Nemeth in order to save switching out of Nemeth just for those numbers. Because they are in the lower part of the cell, the punctuation indicator is required. Nemeth mode is terminated at the end of the last problem that precedes literary material.

Line 12: UEB identifier, UEB bold word indicator.

Keep the switch indicators on the same line as the mathematics to which they apply, except:

a) At the beginning of a list of numbered or lettered identifiers, the opening indicator should be placed either by itself on the line above the first item or at the end of the line of preceding literary material. This ensures that all identifiers begin in the same cell. If space permits, the terminator should be placed on the same line where Nemeth mode ends. See example 15, simbraille lines 8-11.

b) If space permits, an opening Nemeth Code indicator that precedes a spatial problem may be placed on the same line with the end of the text above the problem. The required blank line follows the opening indicator. If there is not room on the line with the preceding text, the indicator may be placed in cell 1 by itself and followed by the required blank line. The spatial problem is followed by a blank line (required according to Nemeth Code) and then the terminator in cell 1 on the following braille line. The blank lines are part of the spatial problem so must be inside the Nemeth switches.
Example 16 (spatial)

Triangle $RST$ has vertices with coordinates $R(-1, -2)$, $S(2, -4)$, and $T(5, 3)$. Find the coordinates of the vertices of this triangle after it is rotated counterclockwise $90^\circ$ about the origin.

Let each column of a matrix represent an ordered pair of the triangle with the top row containing the $x$-values. Then multiply the coordinate matrix by the rotation matrix.

$$
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} \cdot \begin{bmatrix}
-1 & 2 & 5 \\
-2 & -4 & 3
\end{bmatrix} = \begin{bmatrix}
2 & 4 & -3 \\
-1 & 2 & 5
\end{bmatrix}
$$

The coordinates of the vertices of the rotated triangle are $R'(2, -1)$, $S'(4, 2)$, and $T'(-3, 5)$. 
This page intentionally left blank.
Triangle $ABC$ has vertices $A(1, -2)$, $B(2, -4)$, and $C(5, 3)$. Triangle $ABC$ is rotated counterclockwise $90^\circ$ about the origin. 

Diagram

Let the column $R$ of a matrix represent an ordered pair $x$ as triangle $ABC$ top left triangle's $x$-values. Use matrix multiplication to coordinate matrix by a rotation matrix. 

Let coordinates $R$ of vertices $R$ as rotated triangle $R'$ be $B'(2, -1)$, $S'(4, 2)$, and $T'(-3, 5)$. 

Let $A'$, $B'$, and $C'$ be the vertices of triangle $A'B'C'$. 

Diagram
Line 1: "Triangle RST" is the identity of the triangle. The phrase is technical material.

Line 2-3: Commas between math expressions are transcribed in Nemeth Code. A single-word switch indicator before the word "and" means the word is in UEB and is contracted.

Line 6-10: The diagram is technical and will be drawn inside the Nemeth indicators. The blank lines are part of the diagram so are expressed within the indicators. (If the diagram intervenes between two items in Nemeth mode, leave Nemeth in effect for the graphic.)

Line 14: The Opening Nemeth indicator will fit at the end of the last line of text before the spatial material.

Line 16-20: The displayed matrix begins in cell 3 with runovers in cell 5. Enlarged grouping signs enclose the matrices. The numeric indicator is required on numbers in a matrix; there is one blank cell between columns.

Line 22: The terminator is placed at the margin after the required blank line for spatial material.
If exercise directions end with an expression in Nemeth Code and the subsequent problem starts with Nemeth Code, Nemeth may be left in effect between the end of the directions and the start of the problem.

**Example 17 (directions)**

**Find** $f(x + h)$ **for each function** $f(x)$.

34. $f(x) = x + 1$
35. $f(x) = 2x - 3$
36. $f(x) = 4x^2$
37. $f(x) = x^3 - 2x + 5$

---

Line 1: The math expression is transcribed in Nemeth; Nemeth code is terminated at the end of the expression.
Line 2: Nemeth code is initiated for the technical material at the end of the instructions, but is not terminated.
Line 3: Technical material continues in the exercise. Nemeth identifiers are used so that minimal switching is required.
Line 6: Nemeth code is terminated at the end of the last problem.
When short comments in words appear alternated with math problems (such as comments on equation solutions) switch out of Nemeth code to transcribe the comments in contracted braille.

**Example 18 (comments)**

If $y$ varies inversely as $x$, and $y = 3$ when $x = 4$, find $y$ when $x = 18$.

\[
\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \text{Substitute the known values.}
\]

\[
\frac{4}{y_2} = \frac{18}{3}
\]

\[
18y_2 = 12 \quad \text{Cross multiply.}
\]

\[
y_2 = \frac{12}{18} \text{ or } \frac{2}{3} \quad \text{Divide each side by 18.}
\]

The value of $y$ when $x = 18$ is $\frac{2}{3}$.
Example 19 (comments)

Solve \( \cos^2 x - 1 = \sin^2 x \).

\[
\begin{align*}
\cos^2 x - 1 &= \sin^2 x \\
\cos^2 x - 1 &= 1 - \cos^2 x \\
2 \cos^2 x &= 2 \\
\cos^2 x &= 1 \\
\cos x &= \pm 1
\end{align*}
\]

Divide each side by 2.

\[\sin^2 x = 1 - \cos^2 x\]

Take the square root of each side.

\[x = 0^\circ + n \cdot 360^\circ \text{ or } 180^\circ + n \cdot 360^\circ \]

The solutions are \(0^\circ + n \cdot 360^\circ\) and \(180^\circ + n \cdot 360^\circ\) where \(n\) is any integer.

---

Line 1: This is not, by definition, directions. It is a narrative paragraph and starts in cell 3. Line 2 starts in cell 3 as displayed to narrative material.

Line 10: The Opening Nemeth indicator is in the runover position for the comment in order to allow the displayed expression to start in cell 3.

The comments are transcribed on the line following the related expression, indented 4 cells to the right of the runover in effect. This is to eliminate confusion where a comment is also a math expression. This same format must be used throughout the transcription.

Regardless of which method you choose to use, an explanation should be included on the Transcriber's Notes page.
Example 20 (one-word comment)

What percent of 48 is 54?

\[ n \cdot \frac{48}{48} = \frac{54}{48} \]

Write an equation.

\[ \frac{48n}{48} = \frac{54}{48} \]

Divide each side by 48.

\[ n = 1.125 \]

Simplify.

\[ = 112.5\% \]

Change the decimal to a percent.

54 is 112.5% of 48.

Line 1: The freestanding numbers are transcribed in UEB.
Line 2-4: Expressions displayed to narrative text begin in cell 3. The runover margin is cell 5. The author's comments follow the runover margin.
Line 5-8: A linked expression requiring special margins. The runover margin for the anchor and links is cell 7.
Line 6: The author's comment consists of one word. The single-word switch indicator is used on the word "Simplify" even though there are no contractions in the word.
Line 9: New paragraph of the text begins in cell 3. The words "is" and "of" are preceded by the single-word switch indicator. The Opening Nemeth indicator is placed at the beginning of the braille line since the math expression and both indicators will all fit on one line.

The bold type-form for the comments has no meaning and is not retained.
UEB type-form indicators are used in the surrounding text, and Nemeth type-form indicators are used only if it is necessary to indicate emphasis inside the code switches.

If Code switching is necessary within the text of an emphasized passage, such as in a labeled statement, the beginning type-form indicators are repeated after each switch to show that emphasis continues. The effect of a UEB type-form indicator is terminated by the switch from UEB to Nemeth. (Capitalization is not a typeform. A capitals terminator is required to end a capitals passage.) However, a switch from Nemeth to UEB does not terminate the Nemeth emphasis indicators (bold, italic, script). The appropriate type-form indicators (open and close) must be used for emphasized technical material.

To avoid excessive use of indicators, when all labeled statements in a text are printed with the same emphasis, omit the emphasis on the statement unless doing so would change the meaning. The label itself would still be fully capitalized.
### Example 21 (emphasis/labeled statement)

<table>
<thead>
<tr>
<th>Definition of Quadratic Function</th>
<th>A quadratic function is a function described by an equation that can be written in the form ( f(x) = ax^2 + bx + c ) where ( a \neq 0 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Definition of Quadratic Function</td>
</tr>
<tr>
<td>2</td>
<td>Quadratic function is a function described by an equation that</td>
</tr>
<tr>
<td>3</td>
<td>can be written in the form ( f(x) = ax^2 + bx + c ) where ( a \neq 0 ).</td>
</tr>
<tr>
<td>4</td>
<td>[ \text{UEB capitals passage indicator and terminator used on the label of the statement. Nemeth Code requires that the label is fully capitalized.} ]</td>
</tr>
<tr>
<td>5</td>
<td>All labeled statements in the print text are presented in bold type. The typeform has no meaning so is ignored in braille.</td>
</tr>
<tr>
<td>6</td>
<td>Just for comparison the example is shown again with the typeform retained.</td>
</tr>
</tbody>
</table>

Line 1: UEB capitals passage indicator and terminator used on the label of the statement. Nemeth Code requires that the label is fully capitalized.

Line 1-4: All labeled statements in the print text are presented in bold type. The typeform has no meaning so is ignored in braille.

Line 1: UEB capitals passage indicator and terminator.

Line 1: The statement is transcribed as a bold passage.

Line 4: Bold typeform is terminated by the switch to Nemeth. Nemeth bold begin and end indicators enclose the technical material, which is also enclosed within Nemeth switches. UEB font indicators can not be used within the Nemeth switches.

Line 4: Nemeth is terminated before the word "where". The UEB bold word indicator precedes the word "where".

Line 5: Nemeth bold indicators enclose the math expression. The period, which applies to the sentence structure, follows the Nemeth terminator.
Example 22 (partial emphasis)

Newton’s Second Law of Motion

When a net external force $\Sigma F$ acts on an object of mass $m$, the acceleration $a$ that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$a = \frac{\Sigma F}{m} \quad \text{or} \quad \Sigma F = ma$$

In this labeled statement, the emphasis has meaning. The bold letters are vectors. Since the bold is inside the switch indicators, the Nemeth type-form indicators are used. The Nemeth type-form indicator for letters must be followed by an alphabetic indicator.

Line 9, 10: Each of the displayed items starts in cell 3.
If there is not room on a line for the Nemeth Code terminator and any related punctuation, the terminator and punctuation may be separated from the math and placed on the following line.

**Example 23 (placement of switches)**

The probability of getting *exactly* 1850 units of O-negative blood from 32,000 donors is $\binom{32000}{1850} \times 0.06^{1850} \times 0.94^{30150}$. No calculator on earth can calculate that first term.

---

Line 1: UEB italic word indicator

Line 1, 2: Unmodified numbers standing alone are transcribed in UEB.

Line 3: The opening Nemeth indicator is placed at the end of the line of literary text.

Line 4: The expression will fit on one line if the indicators are not on the same line.

Line 5: The terminator and punctuation are placed on the following line.
The Opening Nemeth indicator and the Nemeth Terminator should be placed on the same page with part of the expression to which they apply.

Example 24 (placement of switches)

We have again used the fact that the particles are initially at rest, so that \( v_{01} = v_{02} = 0 \) m/s. Using this result for \( v_{r2} \), we can now obtain the final electric potential energy from the conservation-of-energy equation:

\[
E_{PE_f} = E_{PE_0} - \left[ \frac{1}{2} m_1 v_{r1}^2 + \frac{1}{2} m_2 \left( -\frac{m_1}{m_2} v_{r1} \right)^2 \right]
\]

\[
= E_{PE_0} - \frac{1}{2} m_1 \left( 1 + \frac{m_1}{m_2} \right) v_{r1}^2
\]

\[
= 0.150 \text{ J} - \frac{1}{2} (3.6 \times 10^{-6} \text{ kg}) \left( 1 + \frac{3.6 \times 10^{-6} \text{ kg}}{6.2 \times 10^{-6} \text{ kg}} \right) (170 \text{ m/s})^2
\]

\[
= 0.068 \text{ J}
\]
Line 22: The math expression will fit undivided on the line with both switches on the same line.

Line 1: The Opening Nemeth indicator should be on the same page as the expression to which it applies.

Line 2: EPE is an abbreviation for Electric Potential Energy.

Line 2-10: This is a linked expression that requires special margins: It is displayed, the comparison signs are aligned in print, and all of the links are to the right of the comparison signs. The anchor begins in cell 3, links begin in cell 5, and all runovers to anchor and links are in cell 7.

Line 3, 6: A right numeric subscript to an abbreviation requires a subscript indicator followed by a baseline indicator.

Line 7-10: A division is made between every unit in parentheses.
When a matrix is embedded in text, the opening Nemeth indicator is placed on the top line of the matrix before the opening enlarged grouping symbol on that line; the Nemeth terminator is placed on the top line after the closing enlarged grouping symbol. These indicators apply to the whole arrangement. If there is room on the top line after the matrix, surrounding text can continue on that line.

Example 25 (embedded matrix)

Every square matrix has a determinant. The determinant has the same elements as the matrix, but they are enclosed between vertical bars instead of brackets. In Chapter 3 you learned a method for evaluating a $2 \times 2$ determinant.

The determinant of $\begin{bmatrix} 3 & -2 \\ 17 & 11 \end{bmatrix}$ is $\begin{bmatrix} 3 & -2 \\ 17 & 11 \end{bmatrix}$. To evaluate the determinant, use the rule for second-order determinants.

Line 7-11: Text continues at the top line of the matrix. Nemeth Code is not terminated until the closing of the second embedded matrix.
For a box transcribed all in Nemeth Code, the top box line may include an opening Nemeth Code indicator followed by a space at the beginning of the line. For the bottom box line, the Nemeth Terminator appears at the end of the closing box line preceded by a space. If a transcriber's note occurs inside a box that is otherwise all Nemeth Code, do not include the box lines within Nemeth Code. The UEB TN symbols must be used, and they are not part of the technical code. When possible, place the transcriber's note outside the box.

**Example 26 (table)**

Given the exponent, $x$, compute the power of 2, $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2^x = y$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$2^{-1} = y$</td>
<td>$?$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2^2 = y$</td>
<td>$?$</td>
</tr>
<tr>
<td>$3$</td>
<td>$2^3 = y$</td>
<td>$?$</td>
</tr>
<tr>
<td>$6$</td>
<td>$2^6 = y$</td>
<td>$?$</td>
</tr>
</tbody>
</table>

Given the power of 2, $y$, compute the exponent, $x$.

<table>
<thead>
<tr>
<th>$y$</th>
<th>$2^y = x$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$?$</td>
<td>$2^{y} = \frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$?$</td>
<td>$2^{y} = 4$</td>
<td>$4$</td>
</tr>
<tr>
<td>$?$</td>
<td>$2^{y} = 8$</td>
<td>$8$</td>
</tr>
<tr>
<td>$?$</td>
<td>$2^{y} = 64$</td>
<td>$64$</td>
</tr>
</tbody>
</table>

Line 4, 16: The entire box and its entries are in Nemeth Code. The Opening Nemeth indicator is placed at the beginning of the top box line.

Line 11, 23: The Nemeth Terminator is at the end of the bottom box line.
### Example 27 (table)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>1/4</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>3/4</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>1/3</td>
<td>0.333</td>
<td>33 1/3%</td>
</tr>
<tr>
<td>2/3</td>
<td>0.667</td>
<td>66 2/3%</td>
</tr>
<tr>
<td>1/5</td>
<td>0.2</td>
<td>20%</td>
</tr>
<tr>
<td>2/5</td>
<td>0.4</td>
<td>40%</td>
</tr>
</tbody>
</table>

Line 4: The Opening Nemeth switch is placed at the margin to indicate that everything that follows is in Nemeth Code.

Line 12: Nemeth is closed before the bottom box line.
Example 28 (table)

<table>
<thead>
<tr>
<th>Table 1. Exposure to salinity reduces the growth of wheat plants.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group</strong></td>
</tr>
<tr>
<td>control</td>
</tr>
<tr>
<td>test</td>
</tr>
<tr>
<td>control</td>
</tr>
<tr>
<td>test</td>
</tr>
</tbody>
</table>

Line 3: No blank line between the title and a related box.

Line 4: Column headings are transcribed in UEB.

Line 6: The Opening Nemeth indicator is placed at the margin, indicating that what follows in the table is Nemeth code.

Line 7-8: The row headings are considered to be part of the technical material. The single-word switch indicator is not required. Contractions are not used within Nemeth indicators.

Line 11: Nemeth code terminator follows at the margin.

Line 12: The box lines are not part of the math.
Switches and Formatting

Let’s take a look at applying all of these principles to a page or two from a math book, to see the guidelines in actual practice. This will involve formatting as well as enclosing the math notation in the open and terminate Nemeth switches.

When we talk about format for Nemeth, the formatting issues are those for the technical material and for the surrounding UEB material. The Guidance says that the entire document is arranged according to the accepted compromise between Braille Formats: Principles of Print to Braille Transcription 2011 (for now) and the Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision formatting constrictions. The application of this compromise applies to the entire document, even the text parts that are not within the Nemeth switches. Dot formation is another matter. The only time Nemeth symbols and indicators may be used is within the Nemeth switch indicators.

Nemeth Format

Where the Nemeth Code addresses a formatting issue, this is applied first. Anything that is not addressed in the Nemeth Code will rely on Braille Formats for positioning. Some of the items governed by Nemeth Code format are:

- Division of mathematical expressions
- Narrative paragraph margins of 3-1
- Runover margins for itemized material
- Displayed math expressions
- Instructions
- Exercise with any number of subentry levels
- Table consisting only of numbers
- Blank table entries to be filled in
- Analogies
- Labeled statements
- Rules for keying

Braille Formats

- Displayed literary text
- Indented list for nontechnical text
- Headings
- Any format not covered in the Nemeth Code
There are a few things that will apply to both text and technical material. A most basic example of a Nemeth format applied to the entire document is margins for text. There are no blocked paragraphs in Nemeth Code. All paragraphs begin in cell 3 and runover in cell 1. This applies to the Unified English Braille (UEB) portions of text in a document in which the technical material is transcribed according to the Nemeth Code.

When simple unmodified numbers are mentioned in the text they are transcribed in UEB. However, some formatting rules of the Nemeth Code still apply. Section 195: *A hyphenated expression, of which one component is mathematical, such as "1-inch grid", may not be divided between braille lines. An abbreviation must not be placed on a different braille line from its preceding or following numeral or letter.*

Maria wants to draw a painting with an area of 40 square inches. If she drew something on 1-inch grid paper how many squares would the painting cover?

Martin Luther King, Jr’s birthday is Jan. 15.
Lists and nested lists of itemized material in exercises are formatted according to the Nemeth Code: 1-3 or 1-5, 3-5. In a list with subdivisions, the main item starts in cell 1, runovers in 5. All subdivisions, no matter their depth, begin in cell 3 and runover in cell 5. Runover location is determined individually for each item in a list. Lists of literary material, such as outlines, family trees, and tables of contents, are transcribed according to BF with runovers 2 cells to the right of the deepest indentation. The runover margin for an entire section of literary material would be the same for all items, those with subdivisions and those without.

7. Find the range of each of the following two equations. Show your work.
   a. \( y = 1 + 3 \sin x \)
   b. \( y = 2 + \cos x \)

8. The measures of two angles in a supplementary triangle are in the ratio of 1 : 11. What is the measure of the larger angle?

---

Line 1-4: Problem 7 has subdivisions. Main item is in cell 1; subdivisions begin in cell 3. The runover margin for both main item and subdivisions is cell 5.

Line 2: The Opening Nemeth indicator is placed at the end of the line of text that precedes the technical material.

Line 3, 4: The identifiers a and b are transcribed in Nemeth Code within the switches. The terminator is at the end of the last line of the math.

Line 5-8: Problem 8 has no subdivisions. The runover margin is cell 3.

Line 7: The ratio is transcribed in Nemeth Code. The open indicator, the expression, and the terminator will all fit on one line.
Displayed technical material is transcribed according to Nemeth format.

Technical material that is displayed to narrative text (3-1) begins in cell 3 (two cells to the right of the runover) and runs over in cell 5. No blank lines are inserted preceding or following unless required by other rules of the Code, such as headings or spatial material.

When the material is displayed to itemized text that contains only main divisions (1-3), the displayed material begins in cell 5 (two cells to the right of the runover) and runs over in cell 7. When itemized material contains both main divisions and subdivisions (1-5, 3-5) to whatever depth, a displayed expression begins in cell 7 (two cells to the right of the runover) and runs over in cell 9.

A sentence with an equal sign is called an equation. Here are some examples of equations.

\[ 27 = 9(4 - 1) \quad 1 + 1 = 3 \quad x + 7 = 50 \]

The left equation is true. The middle equation is false. The right equation may be true or false.

Line 3-5: The equations are displayed to narrative material. Each displayed expression begins in cell 3 (two cells to the right of the runover) with runovers, if needed, in cell 5. No blank lines are inserted.

Line 6, 7, 8: Embedded Tns explain the location of left, middle and right.
It is preferred that an exercise not be divided between braille pages whenever possible. At least one line of the instructions must be on the same braille page as the exercise. The text qualifies as an instruction if it applies to a group of numbered or lettered problems that follow. If this is not the case, the text is simple narrative material and is transcribed as a paragraph (3-1). According to Nemeth Code, margins for instructions are 5-3 with additional paragraphs beginning in cell 5 with runovers in cell 3.

Simplify.
41. $32 + 8 \div 4$
42. $24 - 2 \cdot 3 + 6 + 1$
43. $(20 - 8) \cdot 2 + 2$

Evaluate each expression for the given value of the variable.
44. $2(4 + x) - 3$ for $x = 1$
45. $3(8 - x) - 2$ for $x = 2$

46. **Short Response** A company prints $n$ books at a cost of $9$ per book. Write an expression to represent the total cost of printing $n$ books. What is the total cost if 1050 books are printed?

47. **Multiple Choice** Which expression means “3 times the difference of $y$ and 4”?
   - A. $3 \cdot y - 4$
   - B. $3 \cdot (y + 4)$
   - C. $3 \cdot (y - 4)$
   - D. $3 - (y - 4)$
1. Simplify:  

2. Evaluate each expression and give the value for the variable:  

3. Write an expression to represent the total cost if the books are printed.  

4. Write an expression to represent the total cost if 50 books are printed.  

5. Write an expression to represent costs if its digits are reversed.
A math expression should be divided only when it is impossible to get it all on one braille line. Nemeth Code section 195 gives the instructions for dividing long expressions. Besides between items in an enclosed list, the first location for dividing an expression is before a comparison sign. If the expression must also be divided at another point, then division must be made at every comparison sign.

**Solution** Multiplying and dividing by factors of unity, we find the speed limit in feet per second as shown below:

\[
\text{Speed} = \left(\frac{65 \text{ miles}}{\text{hour}}\right) \left(1\right) \left(1\right) = \left(65 \text{ miles} \div \text{hour}\right) \left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) \left(\frac{1 \text{ hour}}{3600 \text{ s}}\right) = 95 \frac{\text{feet}}{\text{second}}
\]
Adding the equations for ball A and ball B gives the following result for the tension in the left half of the rod:

\[ T_B = \frac{mv_B^2}{L} + \frac{mv_A^2}{2L} = \frac{(0.50\,\text{kg})(2.5\,\text{m/s})^2}{0.40\,\text{m}} + \frac{(0.50\,\text{kg})(5.0\,\text{m/s})^2}{2(0.40\,\text{m})} = 23\,\text{N} \]

At the end of the problem set for this chapter, you will find homework problems that contain both conceptual and quantitative parts.

There is a division in the expression at the fraction line (within a unit). Thus it must also be divided at the operation sign and at every comparison sign, which are higher on the priority list. The displayed expression does not require special margins; the comparison signs are not aligned in print.

Line 5+: A dot 5 is required between non-simultaneous subscript and superscript.
Keystroke structures are technical material so must be placed within Nemeth switches.

To add whole numbers on a calculator, we use the `±` and `=` keys. For example, to add 57 and 34, we press `5 | 7 + 3 | 4 =`. The calculator displays `91`, so `57 + 34 = 91`. To find `314 + 259 + 478`, we press `3 | 1 | 4 + 2 | 5 | 9 + 4 | 7 | 8 =`. The display reads `1051`, so `314 + 259 + 478 = 1051`.

Line 5, 9: The window shape symbol shown is not an official part of the Nemeth keystroke rule. The symbol must be listed on the special symbols page.

Line 8-9: Keep a keystroke undivided and use as much of the braille line as possible.
Full Pages from Math Textbooks

1-1 Evaluating Algebraic Expressions

Why learn this? You can evaluate an expression to convert a temperature from degrees Celsius to degrees Fahrenheit. (See Example 3.)

An expression is a mathematical phrase that contains operations, numbers, and/or variables. A variable is a letter that represents a value that can change or vary. There are two types of expressions: numerical and algebraic.

A numerical expression does not contain variables. An algebraic expression contains one or more variables.

<table>
<thead>
<tr>
<th>Numerical Expressions</th>
<th>Algebraic Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 + 2$</td>
<td>$x + 2$</td>
</tr>
<tr>
<td>$27 - 18$</td>
<td>$p - r$</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{x}{4}$</td>
</tr>
</tbody>
</table>

To evaluate an algebraic expression, substitute a given number for the variable. Then use the order of operations to find the value of the resulting numerical expression.

Example 1: Evaluating Algebraic Expressions with One Variable

Evaluate each expression for the given value of the variable.

A $x + 5$ for $x = 11$

$11 + 5$ Substitute 11 for $x$.  
16 Add.

B $2a + 3$ for $a = 4$

$2(4) + 3$ Substitute 4 for $a$.  
8 + 3 Multiply.  
11 Add.

C $4(3 + n) - 2$ for $n = 0, 1, 2$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Substitute</th>
<th>Parentheses</th>
<th>Multiply</th>
<th>Subtract</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4(3 + 0) - 2$</td>
<td>$4(3) - 2$</td>
<td>$12 - 2$</td>
<td>$10$</td>
</tr>
<tr>
<td>1</td>
<td>$4(3 + 1) - 2$</td>
<td>$4(4) - 2$</td>
<td>$16 - 2$</td>
<td>$14$</td>
</tr>
<tr>
<td>2</td>
<td>$4(3 + 2) - 2$</td>
<td>$4(5) - 2$</td>
<td>$20 - 2$</td>
<td>$18$</td>
</tr>
</tbody>
</table>

See Skills Bank p. 5814.

6 Chapter 1 Principles of Algebra
VOCABULARY

expression

variable

numerical expression

algebraic expression

evaluate

Example #1

An expression is a mathematical phrase that states operations, numbers, variables. A variable represents a value that can change or vary. There are two types of expressions: numerical and algebraic.

A numerical algebraic expression does not have variables.


Line 16: Unmodified number, nonmathematical expression transcribed in UEB.

Line 18: First transcriber-defined word indicator representing highlighting. Listed on the SS page. It is not necessary to retain both the bold and the highlighting.
The boxed materials follow the statements to which each applies.

Line 3-10, 16-24: The heading for each box is not mathematical. The opening Nemeth indicator is on the line preceding the technical material in the table. The terminator is placed at the end of the line which contains the final mathematical item. A blank line precedes a top box line and follows a bottom box line. Box lines are no longer (2016) required to be listed on the SS page.
To evaluate an algebraic expression, substitute a given number for variable(s) and use order of operations to find the value. To evaluate numerical expressions:

**Example #2: Evaluate algebraic expressions with variable(s) given.**

1. Evaluate each expression with variable(s) given.

**Value & substitution:** Find the value(s) of each variable.

```
1. a = 5
2. b = 3
3. c = 2
```

4. Interpret what is being done. Use block-cell-cell-column-hidus.

```
- Interpret what is being done. Use block-cell-cell-column-hidus.
- Block: Cell: Cell: Cell: Cell: Column: Hidus. Hidus separate hidus from table entries. A colon separates hidus from:
```

**Line 7-8:** Cell-5 heading

**Line 9-10:** Instructions, margin 5-3

**Line 10:** The Opening Nemeth switch is placed at the end of the line preceding the technical material so that all lettered items start in cell 1.

**Line 12:** Nemeth Code is terminated before the author's comment. The unmodified number and letter are transcribed in UEB.

**Line 11, 14, 18:** The single word switch indicator is used on the word "for" which occurs between two Nemeth expressions.

**Line 19-23:** A tn explains the format for a listed table.
All of the material in the box is technical. Nemeth Code is initiated with the top box line and terminated at the end of the bottom box line.

Listed table format was selected to present the material in the print table. There are too many columns for stairstep format and too many numbers for linear format.
Line 4-9: Second transcriber-defined typeform word indicator is used on PEMDAS. Second transcriber-defined typeform symbol indicator represents the red letter at the beginning of the words in the list. Both indicators must be listed on the SS page.

Line 5-9: Sidebar material is formatted according to its text layout. This is a list with cell-5 and cell-7 headings. List items begin in cell 1.

Line 11: Cross references are 7-5 with a blank line preceding and following. Italics is not retained.
In Example 1 we found the angle when given its sine. The function whose input is the sine of an angle and whose output is the angle is the inverse sine function. Similarly, the inverse cosine and inverse tangent functions pair numbers with angles. Using inverse function notation from algebra, the results of Example 1 are written as
\[
\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ, \quad \cos^{-1}(1) = 0^\circ, \quad \text{and} \quad \tan^{-1}(1) = 45^\circ.
\]
Note that the \(-1\) in this notation does not indicate a reciprocal. Because there are infinitely many angles that have a given sine, cosine, or tangent, we must define these functions precisely. The precise definitions follow.

**Definition: Inverse Sine, Cosine, and Tangent Functions**

- \(\sin^{-1}(x) = \alpha\) provided \(\sin \alpha = x\) and \(-90^\circ \leq \alpha \leq 90^\circ\)
- \(\cos^{-1}(x) = \alpha\) provided \(\cos \alpha = x\) and \(0^\circ \leq \alpha \leq 180^\circ\)
- \(\tan^{-1}(x) = \alpha\) provided \(\tan \alpha = x\) and \(-90^\circ < \alpha < 90^\circ\)

The other three trigonometric functions have inverses also, but we will not need them in studying right triangles. They are discussed in Chapter 4.

**Example 2  Evaluating inverse functions**

Evaluate each expression. Give the result in degrees.

a. \(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\)

b. \(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\)

c. \(\tan^{-1}(\sqrt{3})\)

**Solution**

a. Since \(\cos 45^\circ = \frac{\sqrt{2}}{2}\) and \(0^\circ \leq 45^\circ \leq 180^\circ\), \(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ\).

b. Since \(\sin 60^\circ = \frac{\sqrt{3}}{2}\) and \(-90^\circ \leq 60^\circ \leq 90^\circ\), \(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ\).

c. Since \(\tan 60^\circ = \sqrt{3}\) and \(-90^\circ < 60^\circ < 90^\circ\), \(\tan^{-1}\left(\sqrt{3}\right) = 45^\circ\).

For angles that are not multiples of 30° or 45°, we use a calculator to evaluate an inverse function. Calculators have keys labeled \(\sin^{-1}\), \(\cos^{-1}\), and \(\tan^{-1}\). They are usually the second functions for the \(\sin\), \(\cos\), and \(\tan\) keys. When an inverse function key is pressed, your calculator will give the angle in the range specified in the definition of the inverse functions. Of course the angle will be given in degrees or radians depending on the mode setting.
EXAMPLE 1: WE FIND ANGLE IS 65°.

GIVEN: SIDE1 IS INPUT IS SIDE3.

AN ANGLE IS INPUT IS ANGLE AT 90°.

VISUAL BASIS: FUNCTIONS SIMILARLY.

VIZUE: COSINE AND VIZUE SINE.

FUNCTIONS PAIR NUMBERS TO ANGLES. ETSQ: VIZUE

FUNCTIONS NOTATION ALGEBRAIC RESULTS.

EXAMPLE: 119 WRITTEN 12°.

SINE: 119 WRITTEN 0.945

COSINE: 119 WRITTEN 0.325

TANGENT: 119 WRITTEN 2.906

NOTE: SINE, COSINE AND TANGENT

INDICATE A RECIPROCAL. IT IS DEFINITELY LM ANGLES TO AS GIVE SIDE, COSINE, OR

TANGENT. WE MUST DEFINE AS FUNCTIONS

EXACTLY: AS PRECISE DEFINITIONS FOLLOW.

Line 9, 10, 11: A list of displayed expressions.

Line 10: Single word switch indicator on the word "and".

© NATIONAL BRAILLE ASSOCIATION, INC.
Definition: Use sine, cosine, and tangent functions.

SIN^{-1}(X) \Rightarrow \text{PROVIDE } \sin^{-1}(X)

COS^{-1}(X) \Rightarrow \text{PROVIDE } \cos^{-1}(X)

TAN^{-1}(X) \Rightarrow \text{PROVIDE } \tan^{-1}(X)

At other trigonometric functions, label as we would in math, i.e., study triangles. Example: Evaluate trig functions.

Evaluate each expression. Give results in degrees.

\begin{align*}
\text{A: } \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \\
\text{B: } \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\
\text{C: } \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)
\end{align*}
Solution

1. \[ \cos 30^\circ = 0.866, \sin 30^\circ = 0.5, \tan 30^\circ = \frac{\sqrt{3}}{3} \]

2. \[ \cos 45^\circ = 0.707, \sin 45^\circ = 0.707, \tan 45^\circ = 1 \]

3. \[ \cos 60^\circ = 0.5, \sin 60^\circ = 0.866, \tan 60^\circ = \sqrt{3} \]

4. \[ \cos 90^\circ = 0, \sin 90^\circ = 1, \tan 90^\circ = \text{undefined} \]

5. \[ \cos 180^\circ = -1, \sin 180^\circ = 0, \tan 180^\circ = 0 \]

6. \[ \cos 270^\circ = 0, \sin 270^\circ = -1, \tan 270^\circ = \text{undefined} \]

7. \[ \cos 360^\circ = 1, \sin 360^\circ = 0, \tan 360^\circ = 0 \]

Cross reference

Line 12: Cross reference

Line 14-15: Numbers modified with the degree sign must be transcribed in Nemeth within the switches.
1.3 Angular and Linear Velocity

If the speedometer in your car says 50 mph, then your velocity is 50 mph. Velocity is the rate at which the location of an object is changing with respect to time. If an object is in motion on a circle we can discuss two types of velocity, angular velocity and linear velocity.

Angular Velocity

Consider a point located on a helicopter blade that is rotating at 400 revolutions per minute as shown in Fig. 1.22. As the blade rotates, an angle is formed by the initial and terminal positions of the blade. In one revolution of the blade, the point rotates through an angle of \(2\pi\) radians. The angular velocity of the point is the rate at which the angle is changing. Although 400 revolutions per minute could be considered an angular velocity, we will express angular velocity in radians per unit of time. To find the angular velocity we simply convert 400 rev/min to radians/minute using 
\[
2\pi \text{ rad} = 1 \text{ rev}
\]
and cancellation of units:
\[
\frac{400 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{800\pi \text{ rad}}{1 \text{ min}} = 2513 \text{ rad/min}
\]
So the angular velocity of the point is approximately 2513 rad/min. The angular velocity does not depend on the length of the blade (or the radius of the circular path), but only on the number of revolutions per unit of time.

We use the Greek letter \(\omega\) (omega) to represent angular velocity, and define it as follows.

**Definition: Angular Velocity**

If a point is in motion on a circle through an angle of \(\alpha\) radians in time \(t\), then its angular velocity \(\omega\) is given by
\[
\omega = \frac{\alpha}{t}
\]

Angular velocity can be expressed with many different units of time. For example, an angular velocity can be given as radians per hour, radians per week, radians per year, and so on.

**Example 1 Changing the units**

Convert the angular velocity 240 rad/hr to rad/min.
A. C. TURNANGUL > LIEJ VELOCITY. "FC

IF R SPEEDEMTRY 7 YR CJ SAYS 76 MPH; ON YR VELOCITY IS 76 MPH. VELOCITY IS S RAE AT E S LOCATION R AN OBJECT IS LANGE T RESPECT TO IT. VEIN AN OBJECT IS 3 MOIN ON A CIRCLE WE C .CUSSTWO TYPES R VELOCITY: ANGULR VELOCITY & R IEJ VELOCITY.

ANGULAR VELOCITY

USING A POET LOCATE ON A HELICOPTER BLADE T IS ROTATE AT 400 REVOLUJNS PG MINUTE Y MIN = FIG: JA.22: KINPENURE & A HELICOPTER BLADE & BLADE ROTATES: AN ANGLE IS QM BY 2 INITIAL & TERNAL POSIONS & T BLADE: 1 TO REVOLUJNS R T BLADE. S POET ROTATES BY AN ANGLE R LM 30K RADIANS. WU 11 ANGULAR VELOCITY R S POET IS A RATE AT E S ANGLE IS LANGE. 1 Y REVOLUJNS PG MINUTE CM: USING AN ANGULAR VELOCITY: WE EXPRESS ANGULAR VELOCITY R RADIANS PG UND R IT: TO FED & ANGULAR VELOCITY WE SIMPLY URT EM 100 REV/ MINUTE TO RADIAN S LMINUTE. "USU".

Line 1: Centered heading (3 blank cells on either side)
Line 3: All paragraphs are indented (3-1).
Line 3-10: Linked expression not requiring special margins. Each link fills a line of braille. There are no runovers.
Line 5-7: Fractions containing cancellation must be transcribed spatially. Spatial material requires a blank line preceding and following.
Line 21-25: Labeled statement with displayed expression. Displayed to narrative margins are 3-5. No blank lines are required for displayed technical material.
Line 1: A blank line is required following a labeled statement. If material that requires a blank line ends on line 24 or 25, the blank line should be left at the top of the next braille page.